Conformal Refinement and Coarsening of Unstructured Quadrilateral Meshes

Rao V. Garimella, T-5

We have developed a novel method of adapting unstructured quadrilateral (quad) meshes for accurately capturing special features of unsteady solutions of partial differential equations, or PDEs. Adaptive refinement enables the capturing of complex solution features by focusing refinement in critical areas without having to refine the mesh everywhere. In our method, we can refine or coarsen unstructured quadrilateral meshes as driven by an error indicator or estimator while maintaining a conformal mesh (meaning no node of a refined quadrilateral lies inside an edge of a coarser quadrilateral). This implies that a numerical method that works on a fixed conformal mesh can be applied as-is to dynamically adapted meshes.

The adaptive mesh modification algorithm starts with tagging elements that must be refined because they do not adequately represent some geometric feature or because the solution error in these elements is too high. These elements and their edges are tagged for refinement (or coarsening), if necessary, to multiple levels below (or above) their current level of refinement. Once the appropriate elements have been tagged by the application, the mesh is coarsened wherever the elements are smaller than they need to be. After coarsening, the mesh is refined wherever the elements are larger than they need to be. During both coarsening and refinement, the target refinement levels of elements are adjusted so that they are consistent with their siblings (children of their parents) and such that the target refinement levels of two adjacent elements do not differ by more than one. The one-level difference rule ensures that the number of templates required to make the mesh conforming is limited to a manageable number and that the mesh is smoothly graded.

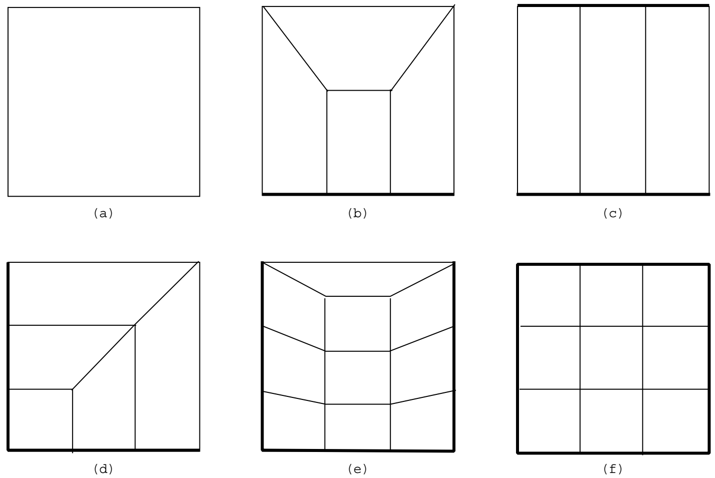

Fig. 2. Subdivision templates (thick edges are refined) (a) unrefined quad (b) one edge refined (c) two opposite edges refined (d) two adjacent edges refined (e) three edges refined (f) all edges refined (uniform refinement).

Whenever a quad is uniformly refined, edges of adjacent quads are also refined. To make the mesh strictly conforming, these adjacent elements must also be subdivided into quadrilaterals such that there are no nonconforming nodes (Fig. 1). The templates used for subdividing quadrilaterals with different edges refined are shown in Fig.2. Some of these templates have been described in previous works [1] and some are new.

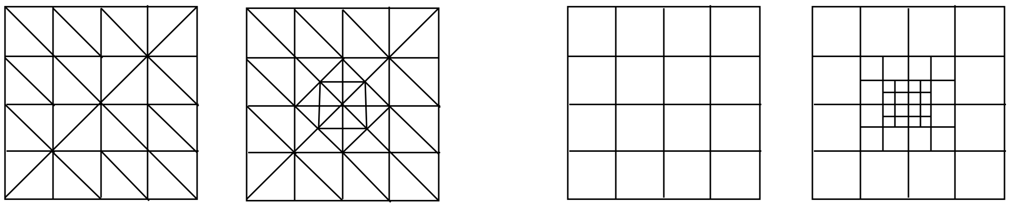

Fig. 1. Conformal refinement of triangles (left) and non-conformal refinement of quads (right).

The quads that result from uniform refinement of a parent quad are called regular elements, while quads resulting from refinement of one, two, or three edges of the parent quad are called irregular quads. An important aspect of this method is that irregular quads are never refined since their subdivision can result in substantial degradation of quality. Rather, if an irregular quad is considered to be too large, it is deleted along with its siblings, and the parent is refined to the required size.

First we show examples of a structured mesh and an unstructured mesh adapted to a superimposed line in the mesh (Fig 3). Any element that is intersected by the line is refined up to level 3 (level 0 is the original mesh). The superimposed line goes from (–0.308207,1.106007) to (1.106007 – 0.308207)(Fig.4).

Next, two snapshots (Fig. 5) of a dynamic adaptation procedure are shown in which elements intersecting one of two expanding circles are refined to level 3 and elements intersecting both circles are refined to level 4. One circle is centered at (0.0,0.0), and the other circle is centered at (1.0,0.25). Both circles start with a radius of 0.11 with their radii increasing in increments of 0.05. As the circles grow, they intersect each other and eventually grow out of the domain.

For more information contact Rao V. Garimella at rao@lanl.gov.

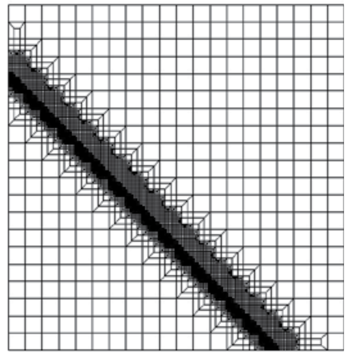

Fig. 3. A 20x20 structured mesh refined along a line.

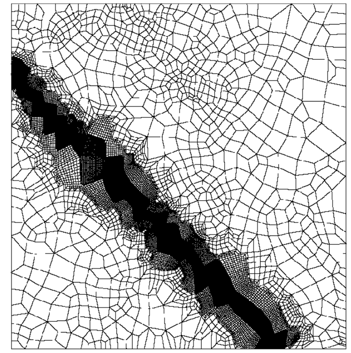

Fig. 4. An unstructured mesh refined along the same line.

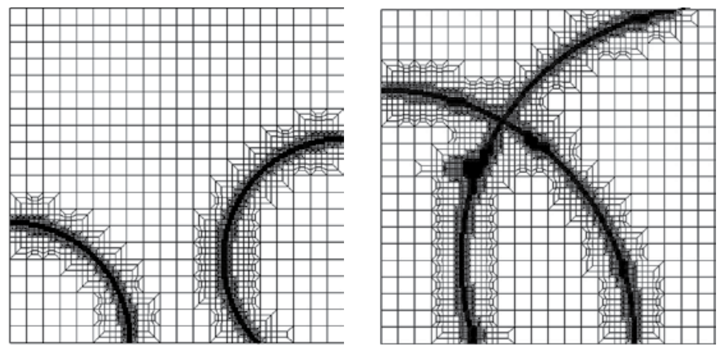

Fig. 5. Snapshots of dynamic mesh adaptation of a 20x20 structured mesh with respect to two expanding circles

[1] J.S. Sandhu et al., *Eng. Fract. Mech.* **50,** 727 (1995).

Funding Acknowledgments
LANL Directed Research and Development Program